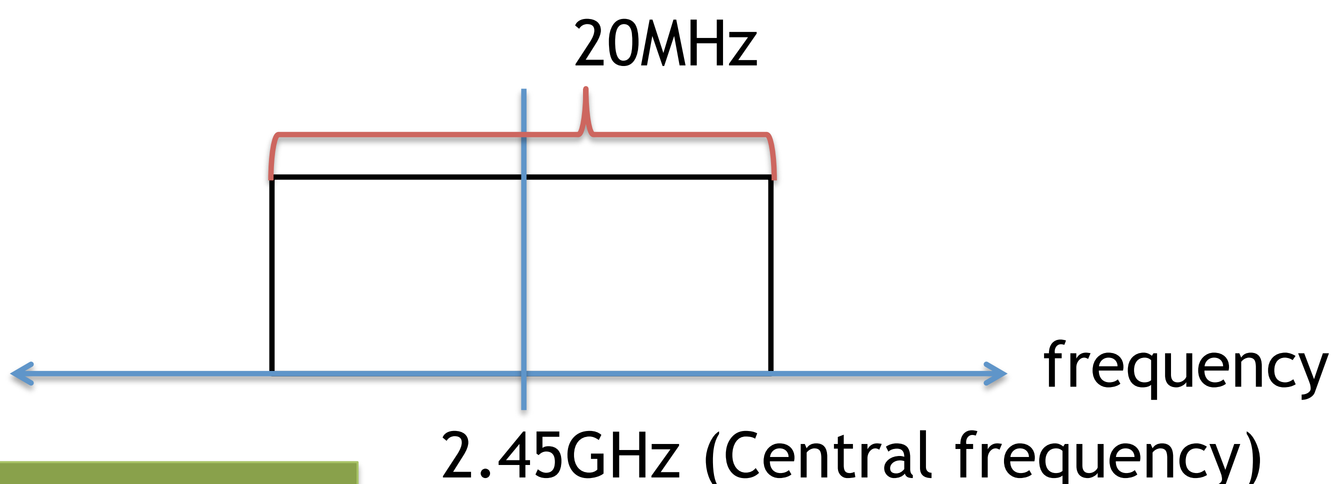
Orthogonal Frequency Division Modulation (OFDM)

- OFDM diagram
- Inter Symbol Interference
- Packet detection and synchronization
- Related works

Motivation

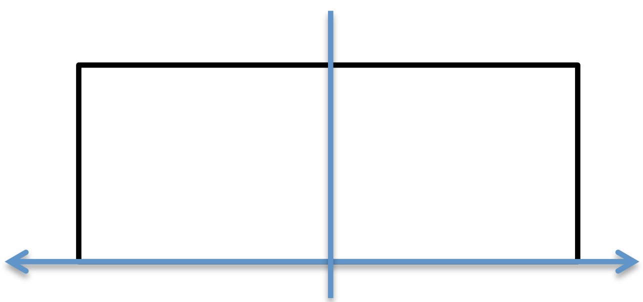
- Signal over wireless channel
  - y[n] = Hx[n]
- Work only for narrow-band channels, but not for wide-band channels
  - e.g., 20 MHz for 802.11



Capacity = BW * log(1+SNR)

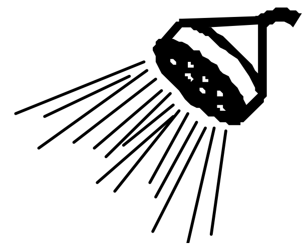
Basic Concept of OFDM

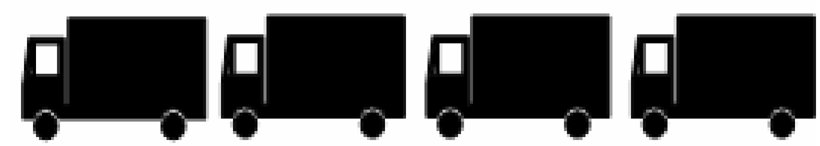
Wide-band channel

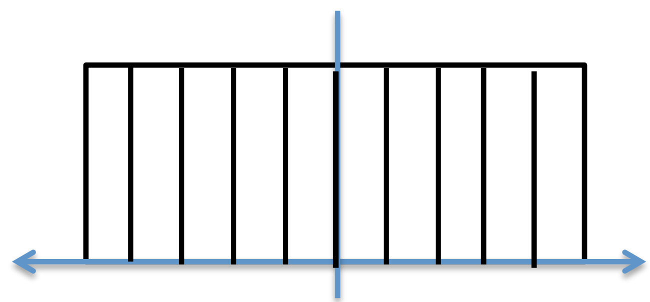


Send a sample using the entire band

Multiple narrow-band channels

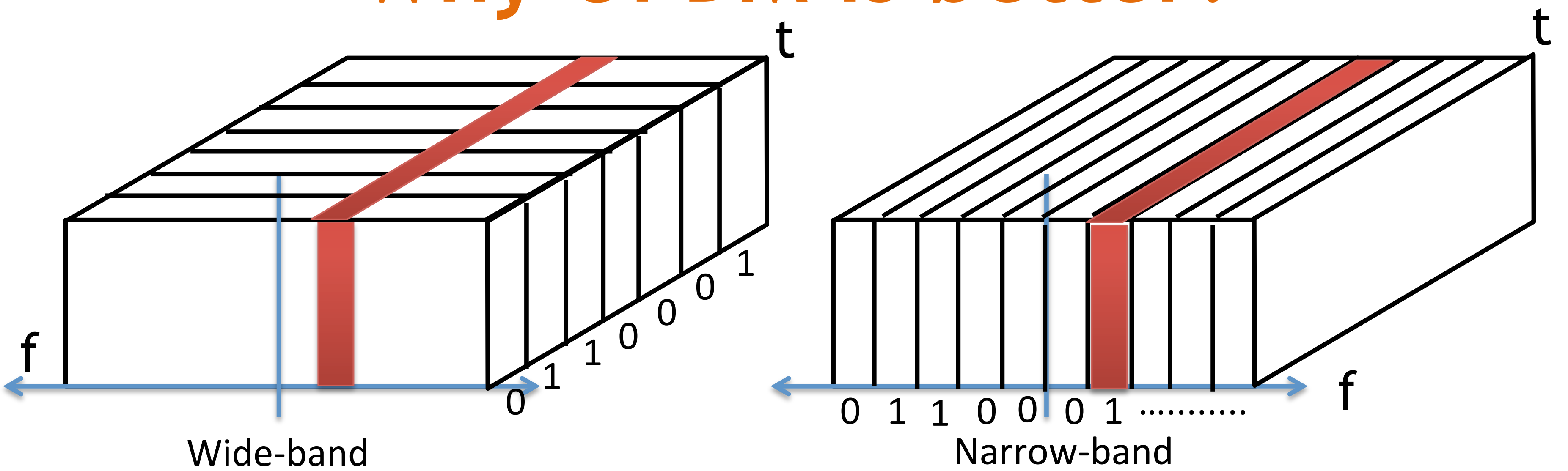






Send samples concurrently using multiple **orthogonal sub-channels**

Why OFDM is better?



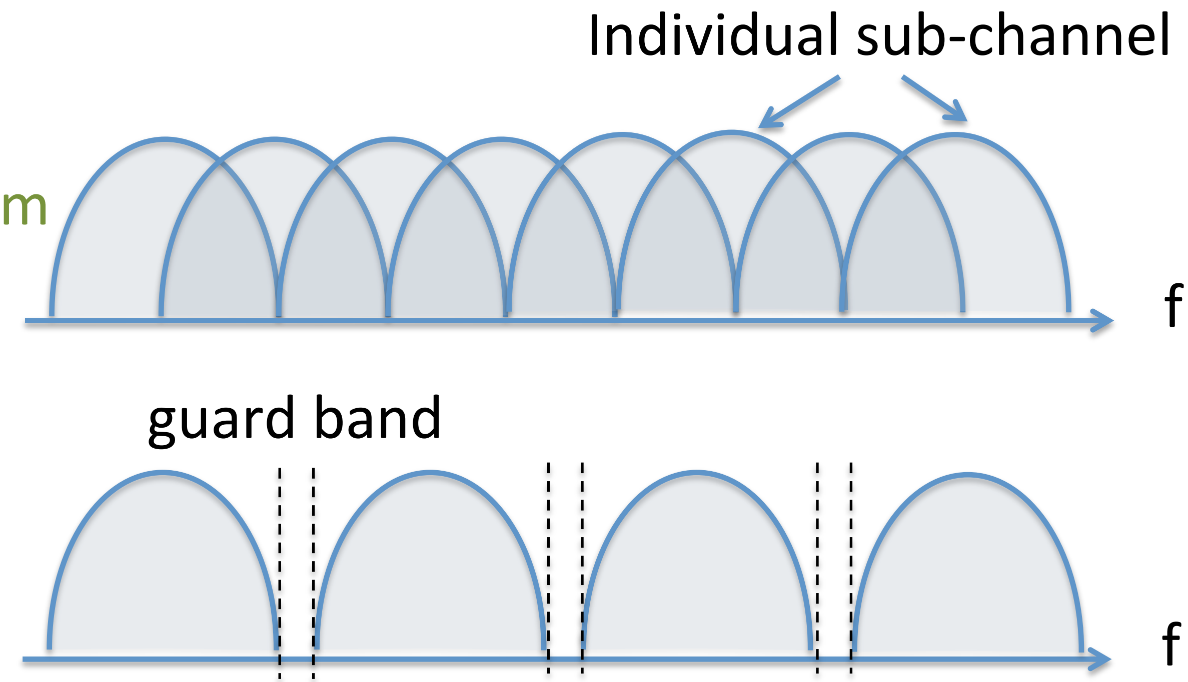
- Multiple sub-channels (sub-carriers) carry samples sent at a lower rate
  - Almost same bandwidth with wide-band channel
- Only some of the sub-channels are affected by interferers or multi-path effect

Importance of Orthogonality

- Why not just use FDM (frequency division multiplexing)
    - Not orthogonal

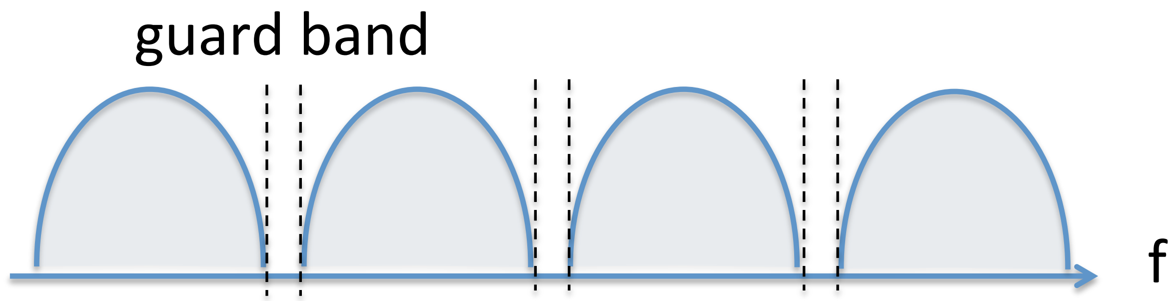
Leakage interference from adjacent sub-channels

Guard bands protect leakage interference

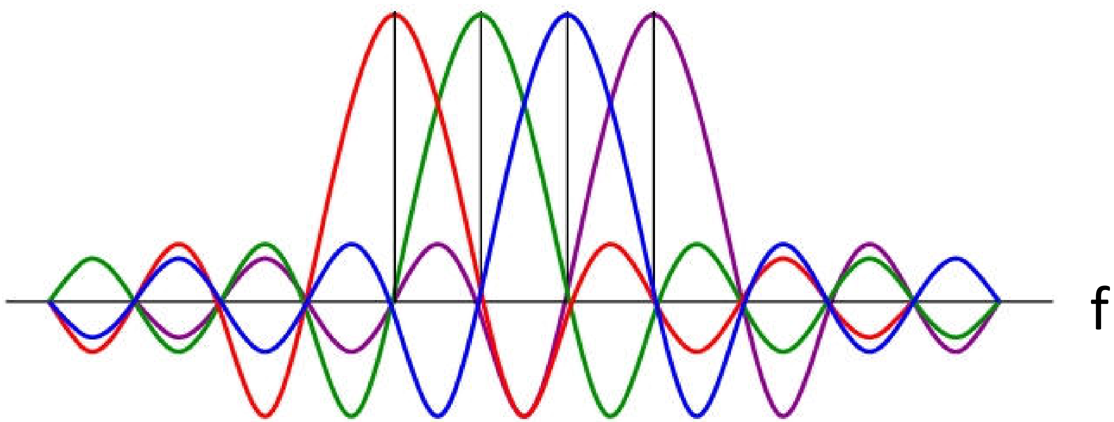


- Need guard bands between adjacent frequency bands → extra overhead and lower throughput

Difference between FDM and OFDM



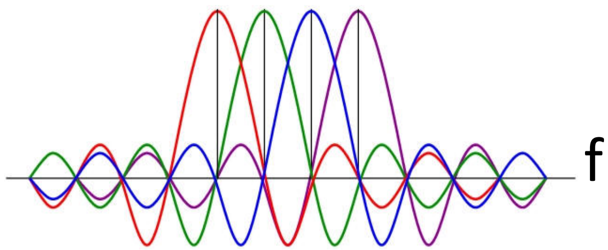
Frequency division multiplexing



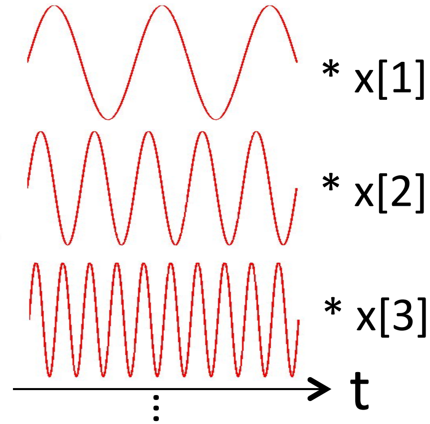
Orthogonal sub-carriers in OFDM

Don't need guard bands

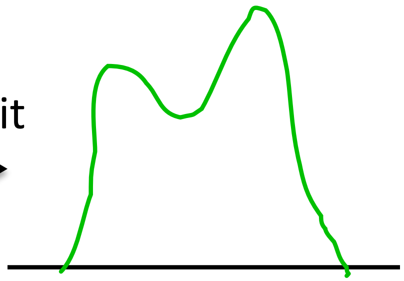
Orthogonal Frequency Division Modulation



IFFT →



transmit →

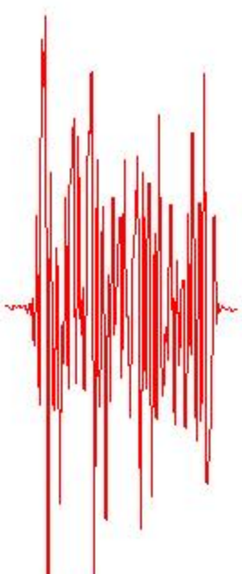


Data coded in frequency domain

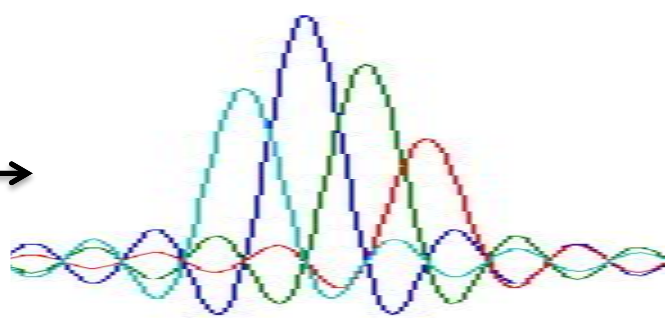
Transformation to time domain: each frequency is a sine wave
In time, all added up

Channel frequency response

receive →



FFT →

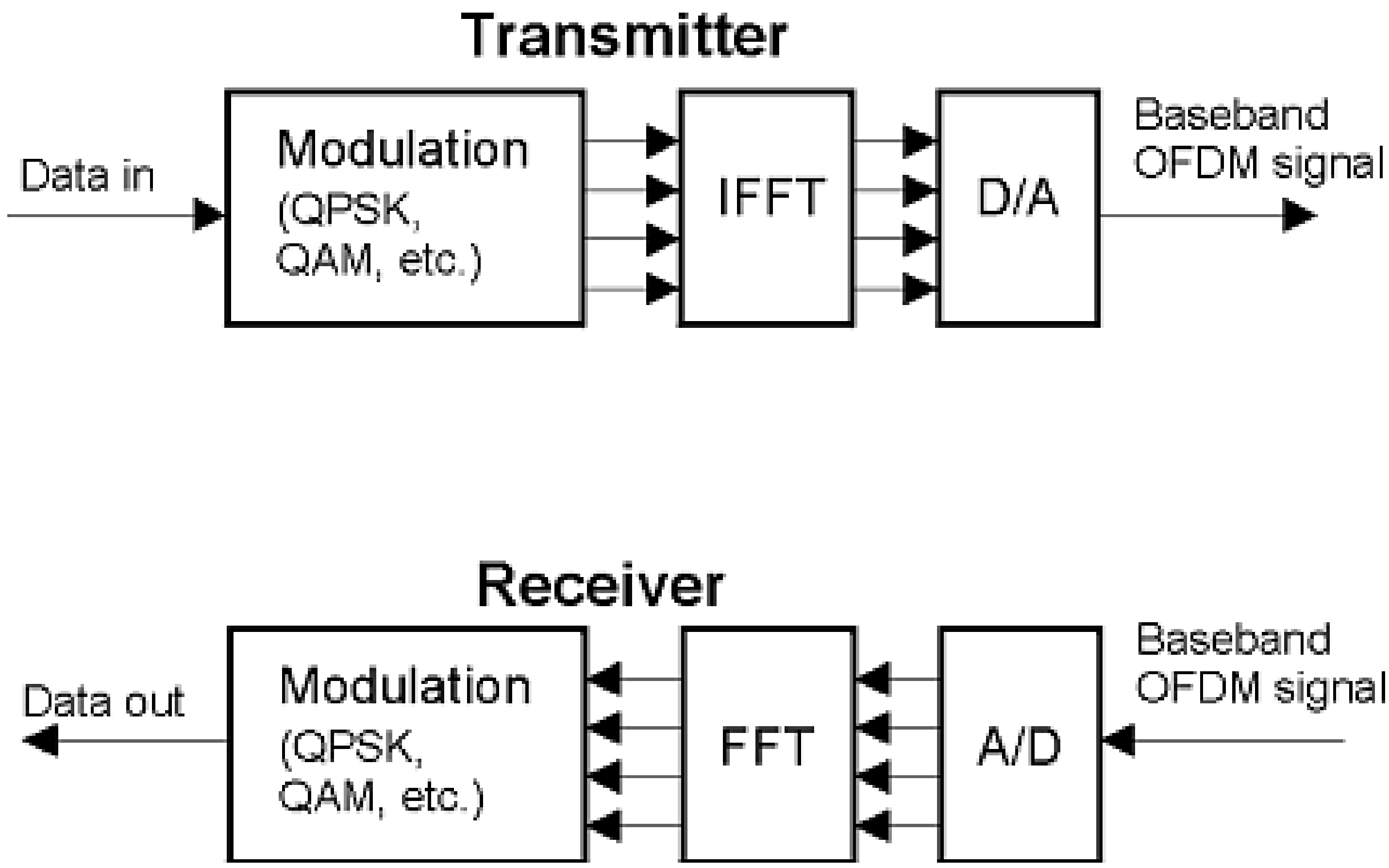


Decode each subcarrier separately →

Time domain signal

Frequency domain signal

OFDM Transmitter and Receiver



Orthogonality of Sub-carriers

IFFT

Encode: frequency-domain samples → time-domain sample

$$x(t) = \sum_{k=-N/2}^{N/2-1} X[k] e^{j2\pi kt/N}$$

Time-domain

Frequency-domain

$$X[k] = \frac{1}{N} \sum_{t=N/2}^{N/2-1} x(t) e^{-j2\pi kt/N}$$

FFT

Decode: time-domain samples → frequency-domain sample

Orthogonality of any two bins : $\sum_{t=N/2}^{N/2-1} e^{-j2\pi kt/N} e^{-j2\pi pt/N} = 0, \forall p \neq k$

Example

- Say we use BPSK and 4 sub-carriers to transmit a stream of samples

1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1,…

- Serial to parallel conversion of samples

Frequency-domain signal

	c1	c2	c3	c4
symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1

IFFT →

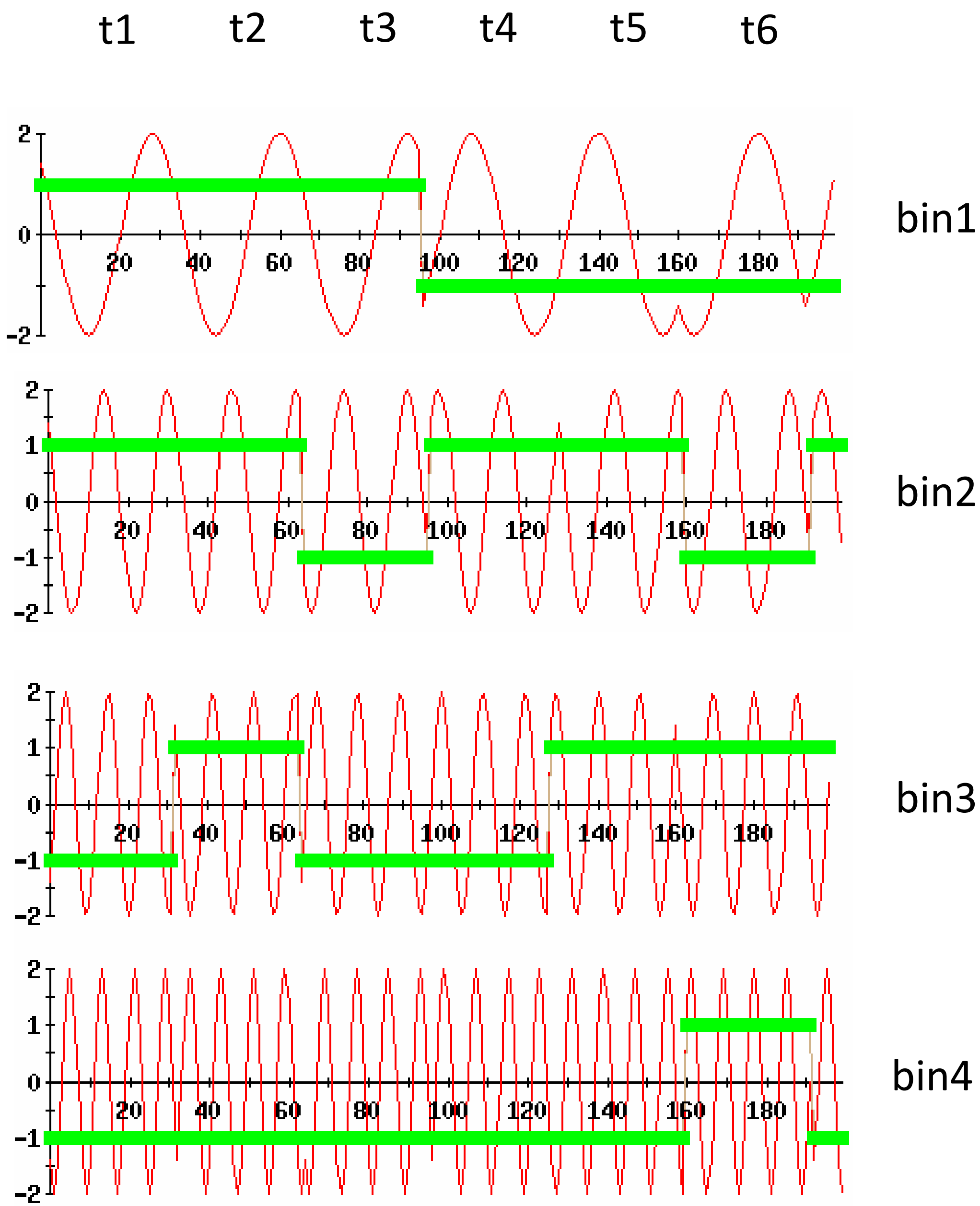
Time-domain signal

0	2 - 2i	0	2 + 2i
2	0 - 2i	2	0 + 2i
-2	2	2	2
-2	0 - 2i	-2	0 + 2i
0	-2 - 2i	0	-2 + 2i
0	-2 + 2i	0	-2 - 2i

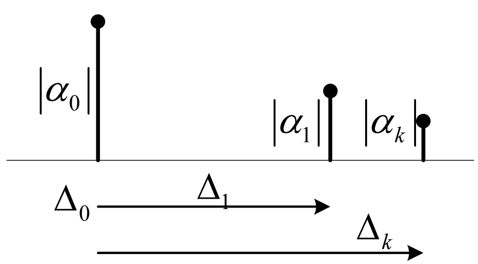
- Parallel to serial conversion, and transmit time-domain samples

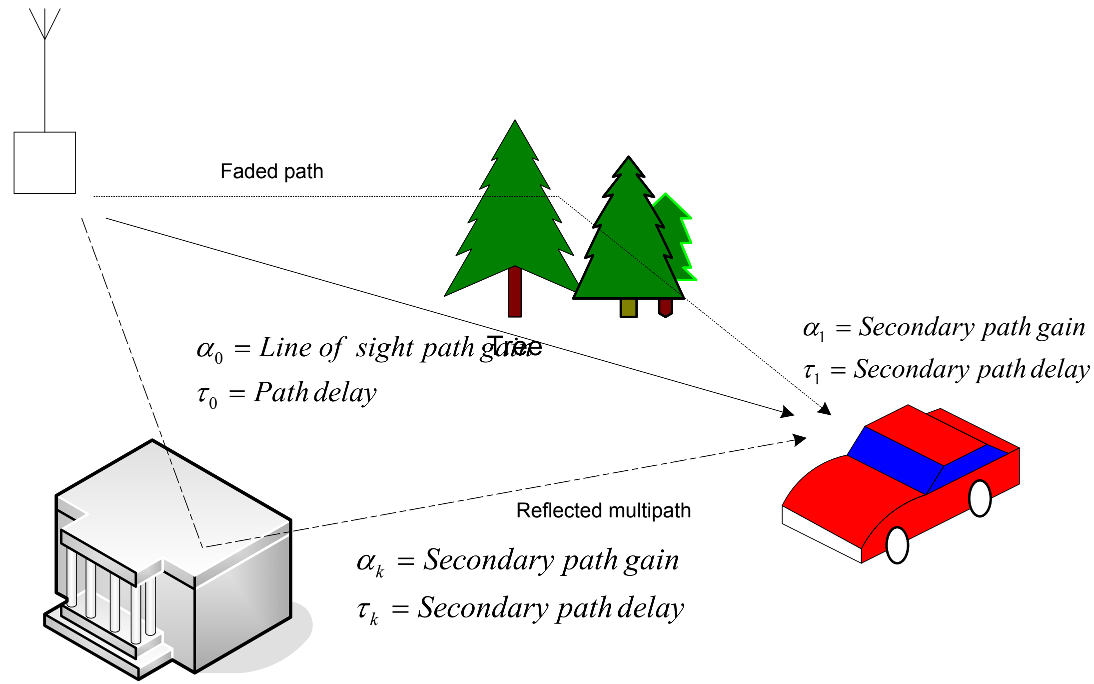
0, 2 - 2i, 0, 2 + 2i, 2, 0 - 2i, 2, 0 + 2i, -2, 2, 2, 2, -2, 0 - 2i, -2,
0 + 2i, 0, -2 - 2i, 0, -2 + 2i, 0, -2 + 2i, 0, -2 - 2i, …

symbol1	1	1	-1	-1
symbol2	1	1	1	-1
symbol3	1	-1	-1	-1
symbol4	-1	1	-1	-1
symbol5	-1	1	1	-1
symbol6	-1	-1	1	1



Multi-Path Effect

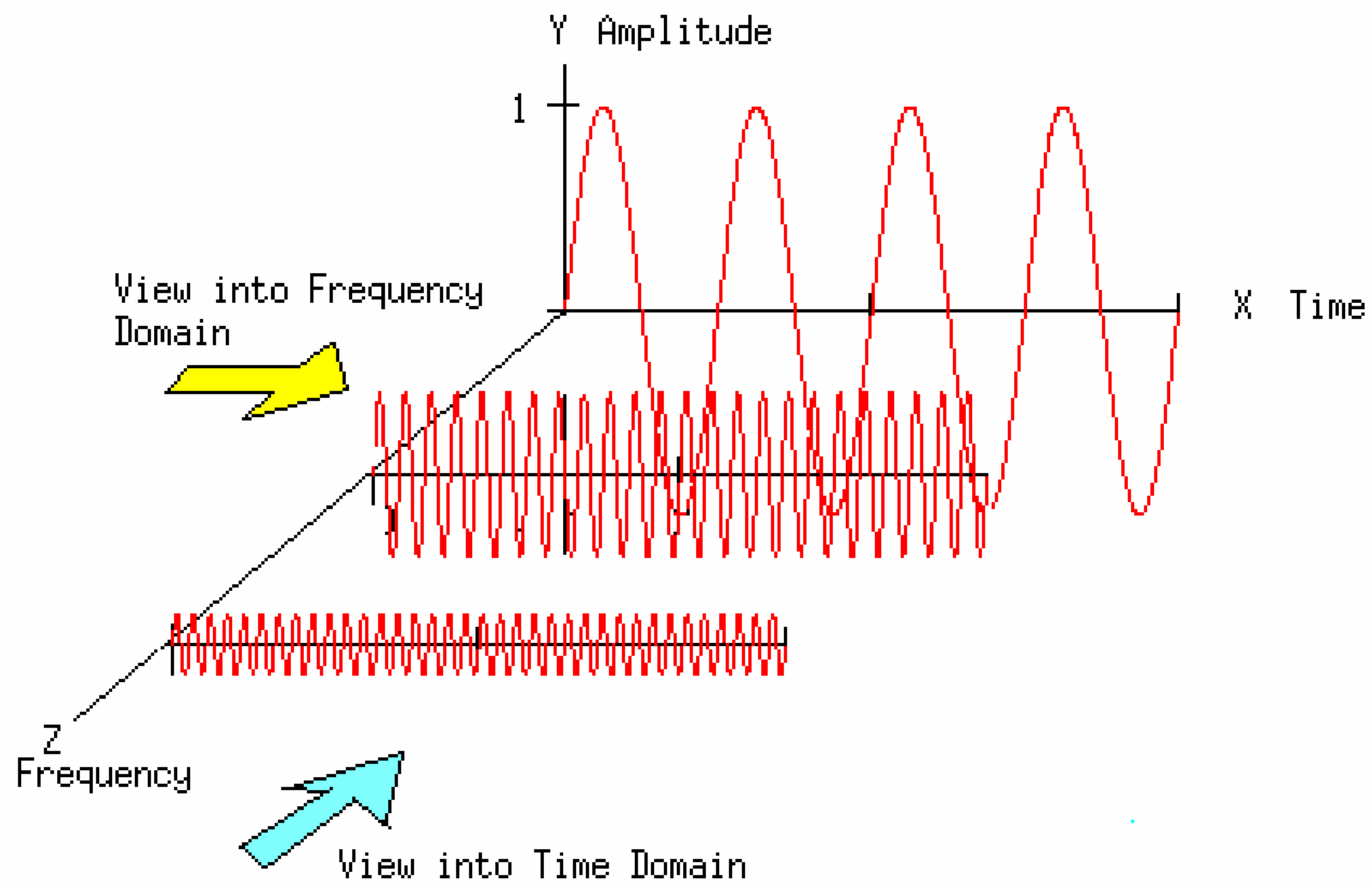




$$y(t) = h(0)x(t) + h(1)x(t-1) + h(2)x(t-2) + \cdots$$
$$= \sum_{\Delta} h(\Delta)x(t-\Delta) = h(t) \otimes x(t) \quad \Leftrightarrow \quad Y(f) = H(f)X(f)$$

time-domain

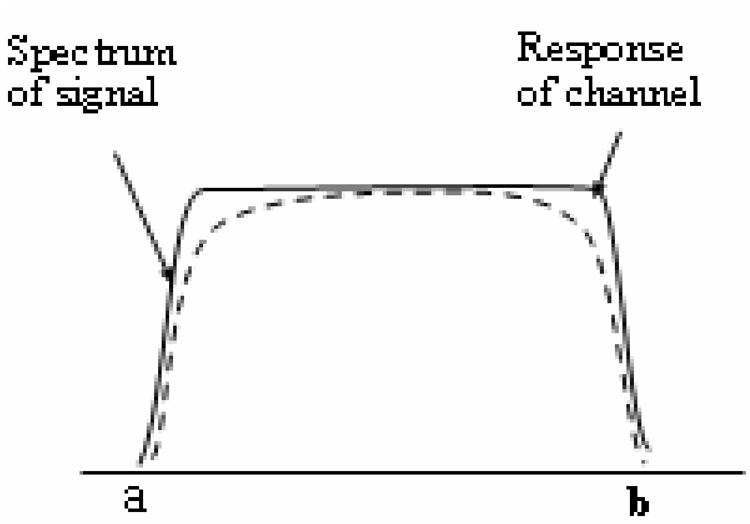
frequency-domain

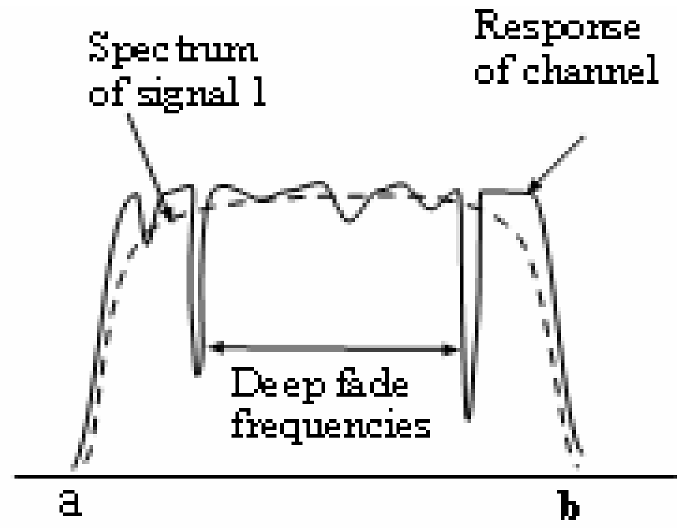


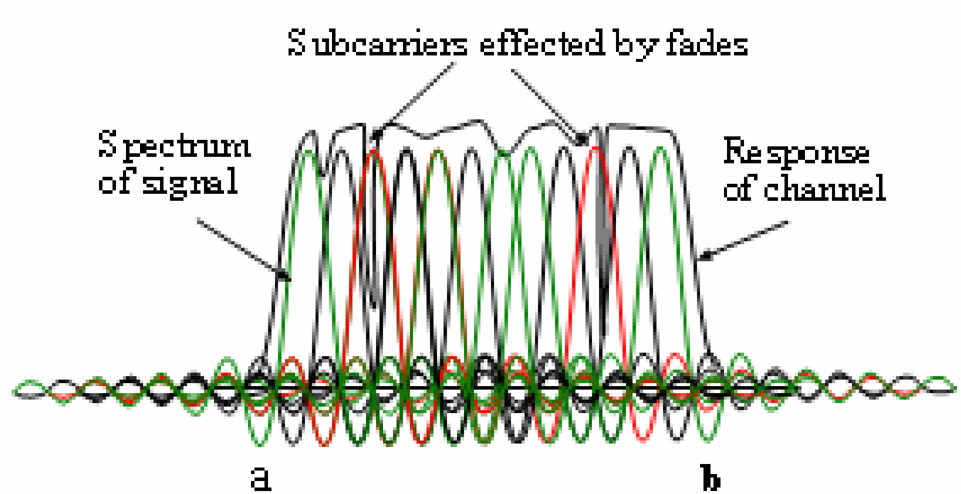
Current symbol + delayed-version symbol

→ Signals are deconstructive in only certain frequencies

Frequency Selective Fading



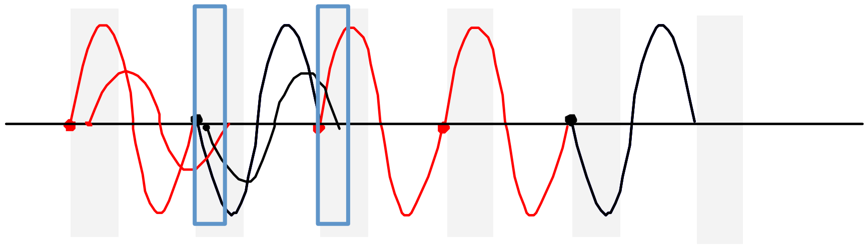




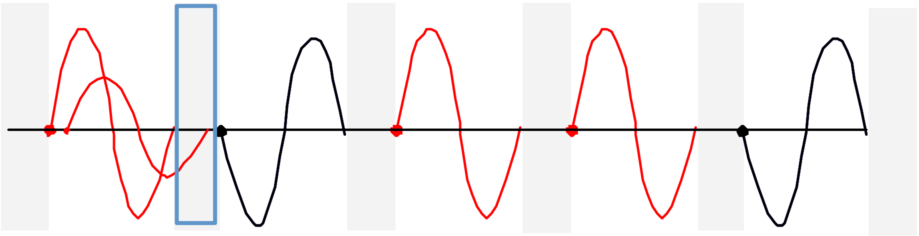
Frequency selective fading: Only some sub-carriers get affected

Inter Symbol Interference (ISI)

- The delayed version of a symbol overlaps with the adjacent symbol



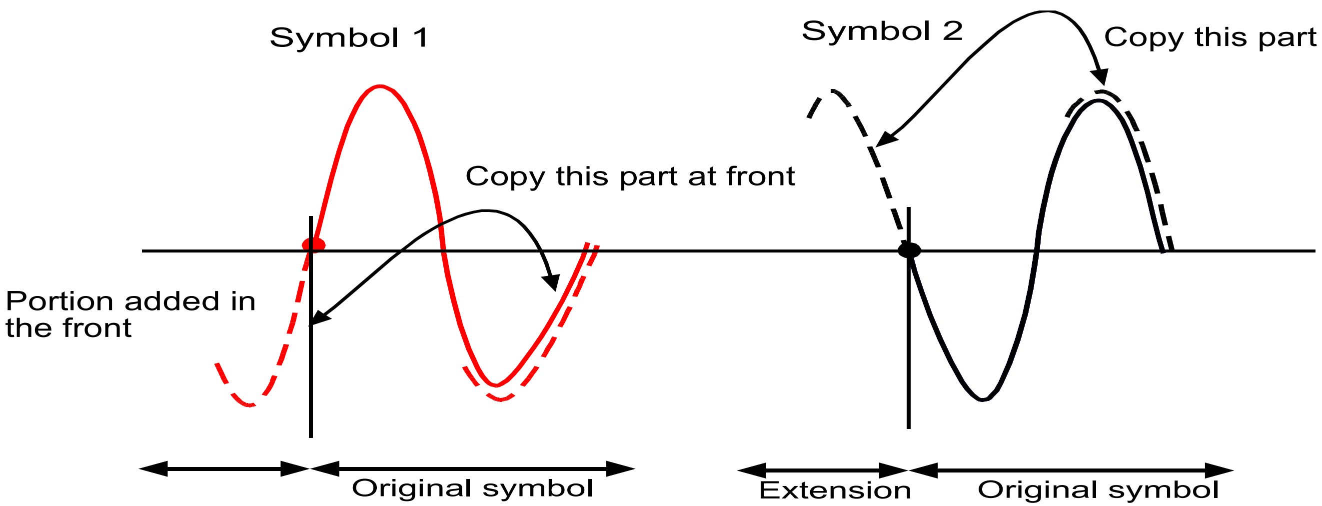
- One simple solution to avoid this is to introduce a guard-band



Guard band

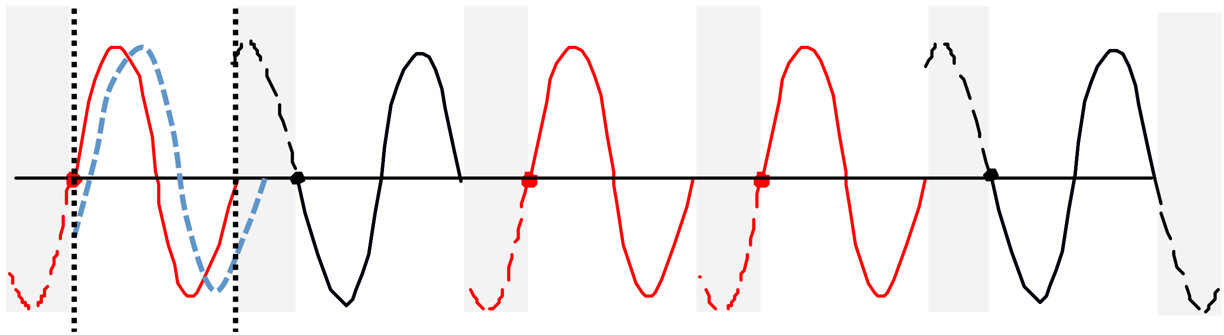
Cyclic Prefix (CP)

- However, we don't know the delay spread exactly
  - The hardware doesn't allow blank space because it needs to send out signals continuously
- Solution: Cyclic Prefix
  - Make the symbol period longer by copying the tail and glue it in the front

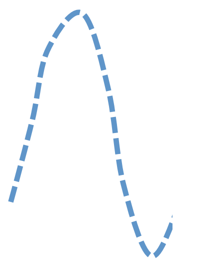
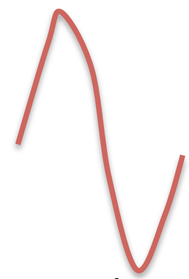


In 802.11,
CP:data = 1:4

Cyclic Prefix (CP)



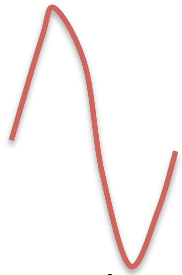
- Because of the usage of FFT, the signal is periodic

FFT() = exp(-2jπ_{Δ}f)*FFT()

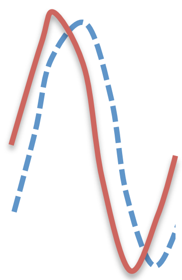
delayed version — original signal

- Delay in the time domain corresponds to rotation in the frequency domain
  - Can still obtain the correct signal in the frequency domain by compensating this rotation

Cyclic Prefix (CP)

w/o multipath $y(t) \rightarrow FFT($  $) \rightarrow Y[k] = H[k]X[k]$

original signal

w multipath $y(t) \rightarrow FFT($  $) \rightarrow Y[k] = \alpha(1+\exp(-2j\pi_{\Delta}k))*X[k]$

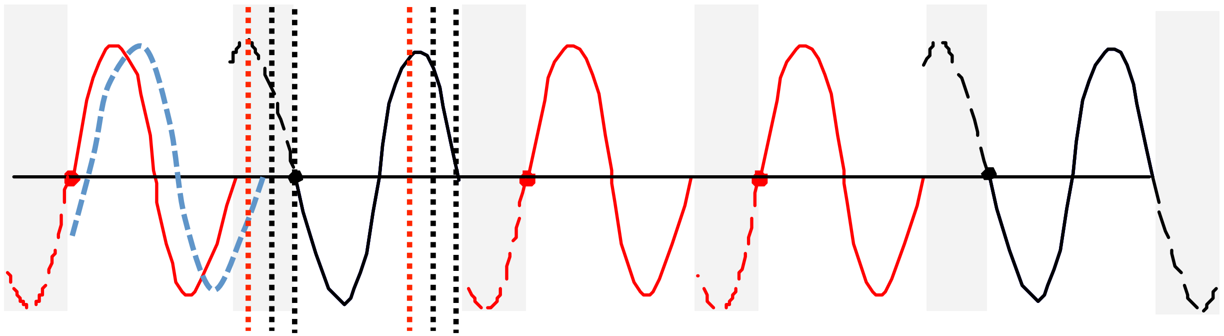
$= H'[k]X[k]$

original signal + delayed-version signal

Lump the phase shift in H

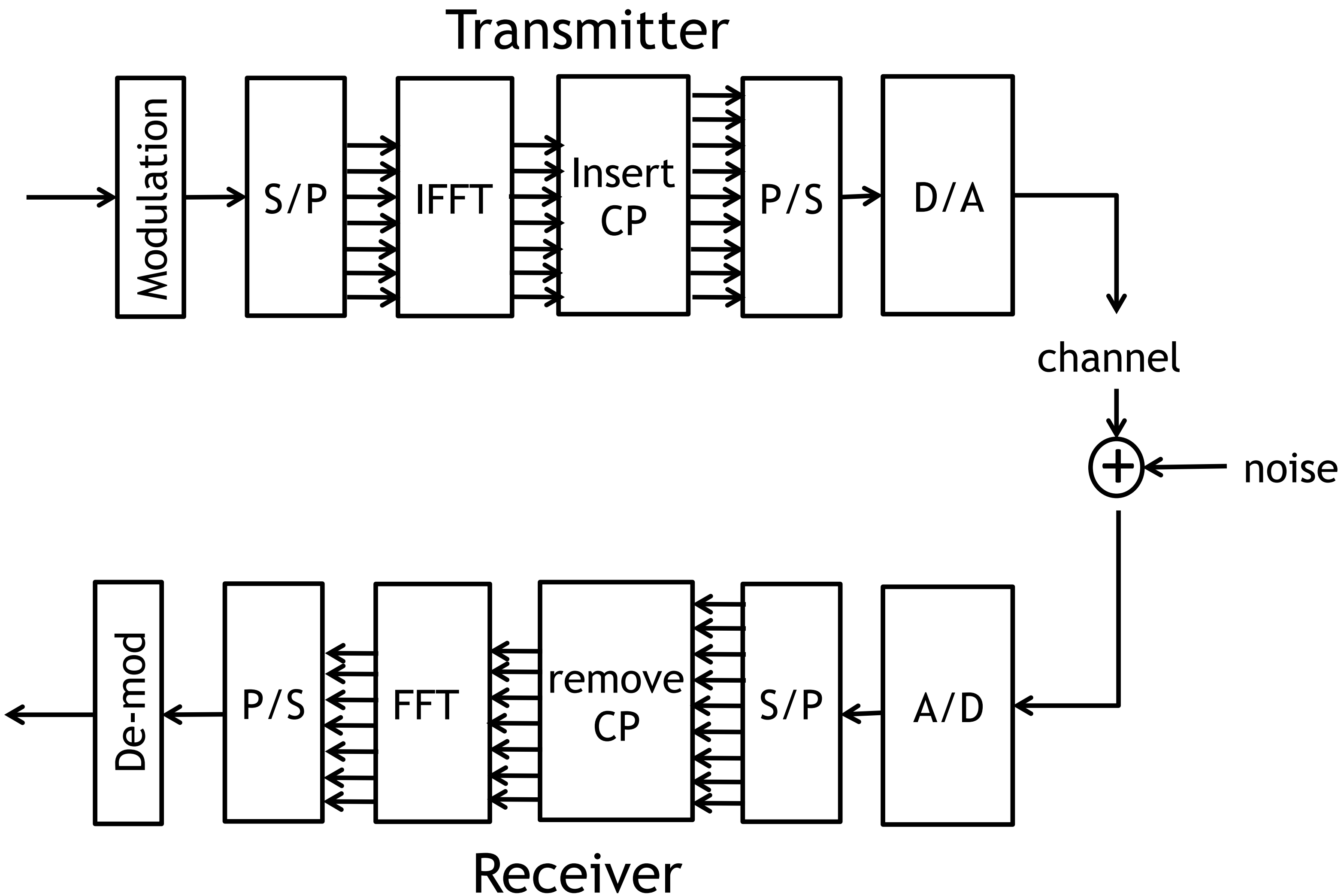
Side Benefit of CP

- Allow the signal to be decoded even if the packet is detected after some delay

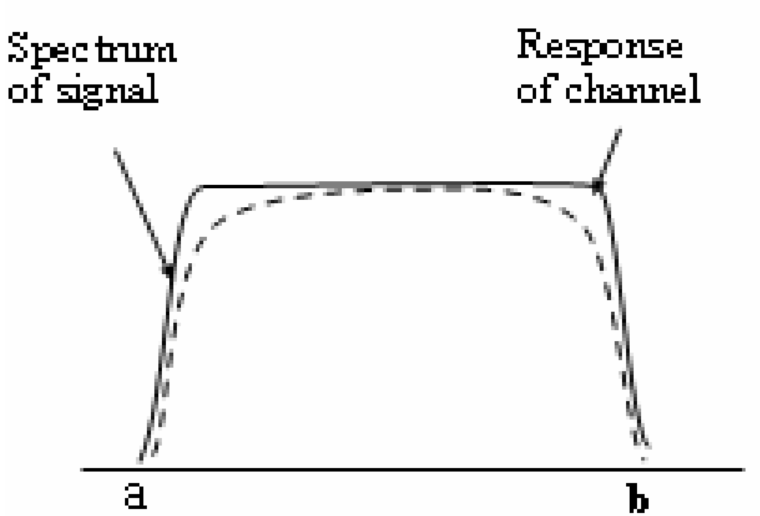


un**decodable**

OFDM Diagram

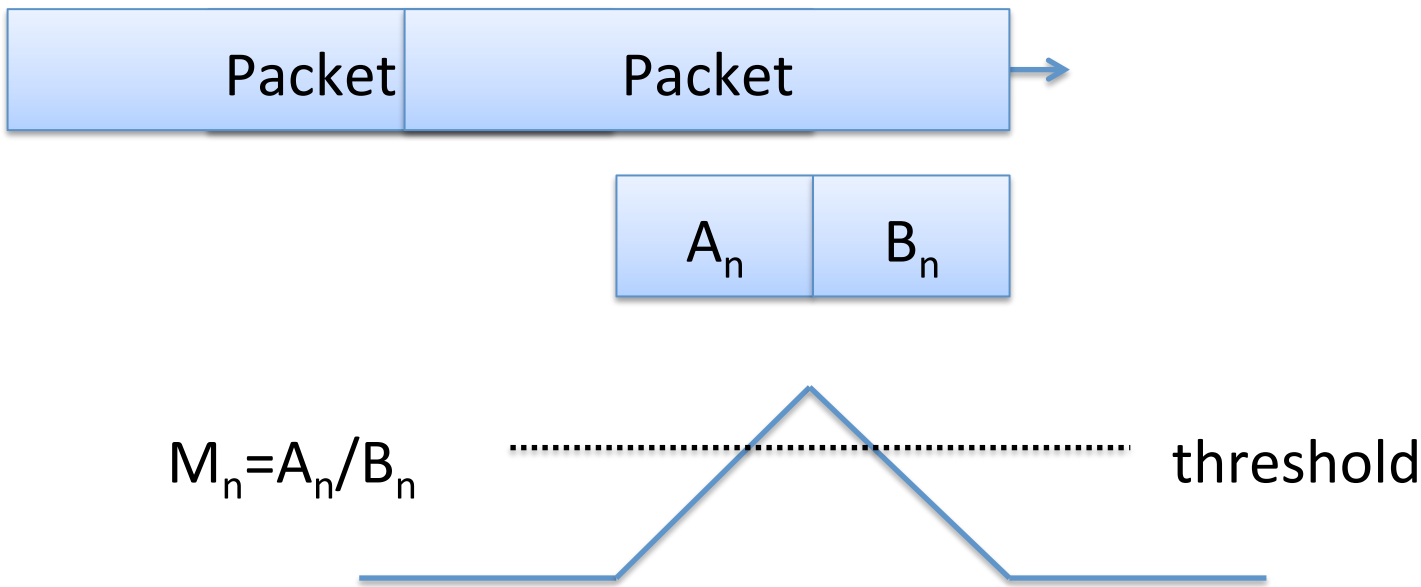


Unoccupied Subcarriers



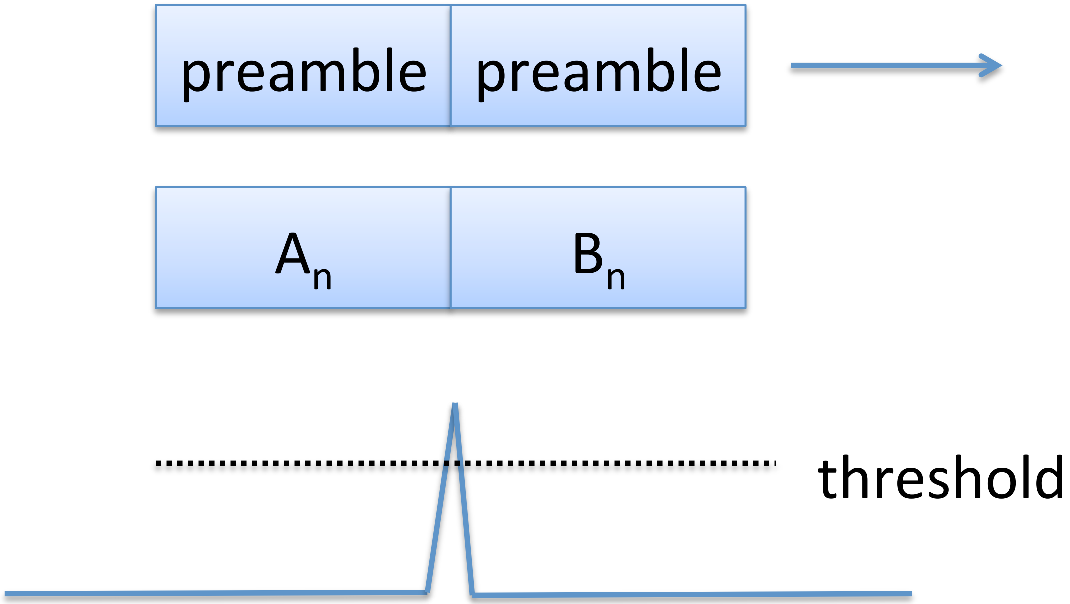
- Edge sub-carriers are more vulnerable to errors under discrete FFT
  - Frequency might be shifted due to noise or multi-path
- Leave them unused
  - In 802.11, only 48 of 64 bins are occupied bins
- Is it really worth to use OFDM when it costs so many overheads (CP, unoccupied bins)?

Packet Detection



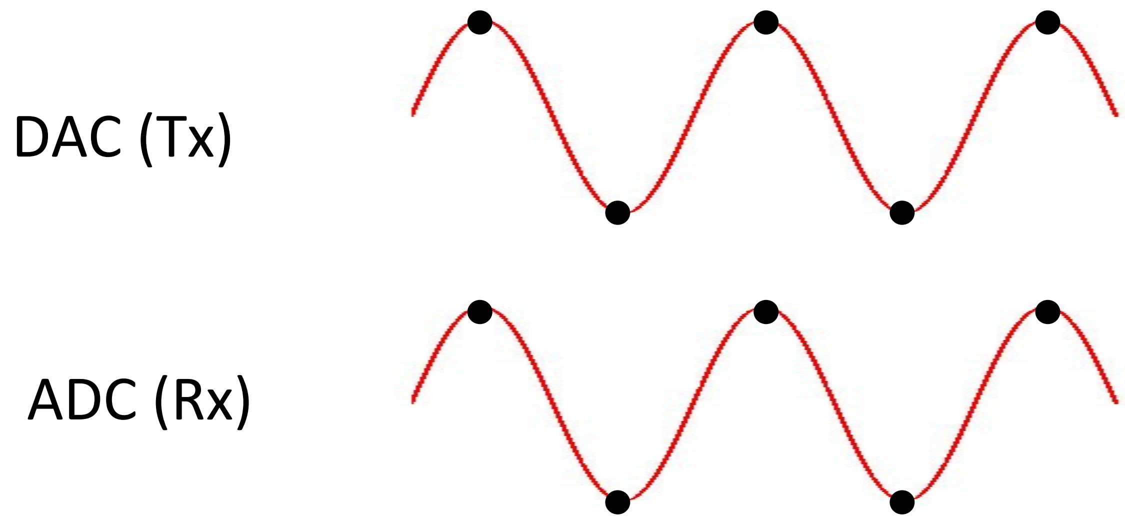
- Double sliding window packet detection
- Optimal threshold depends on the receiving power

Packet Detection



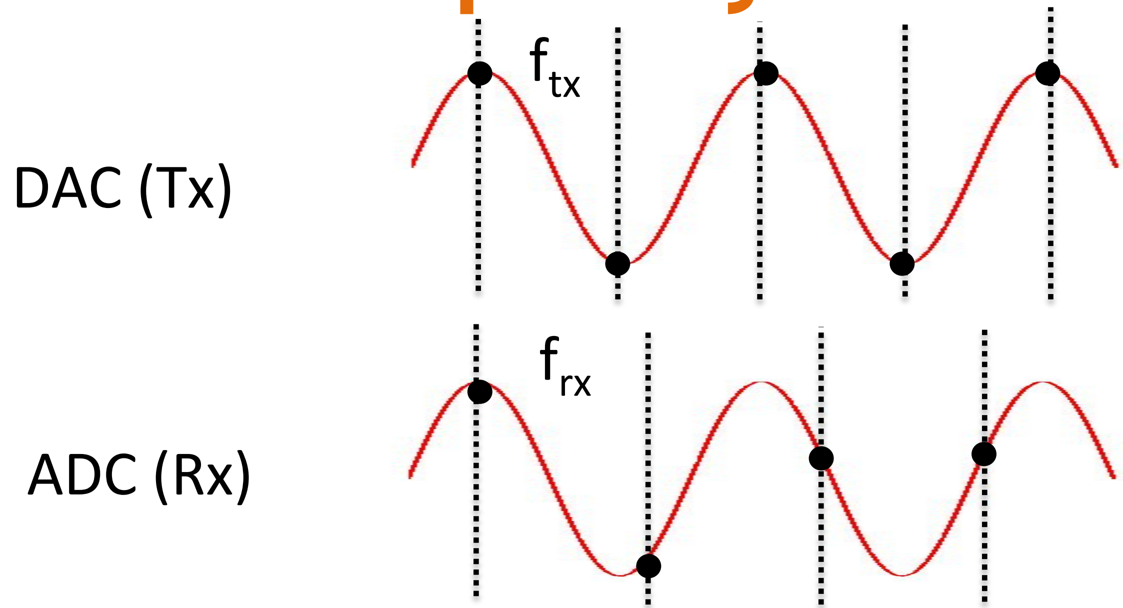
- Use cross-correlation to detect the preamble

Synchronization



- DAC (at Tx) and ADC (at Rx) never have exactly the sampling period
  - A slow shift of the symbol timing point, which rotates subcarriers
  - Intercarrier interference (ICI), which causes loss of the orthogonality of the subcarriers

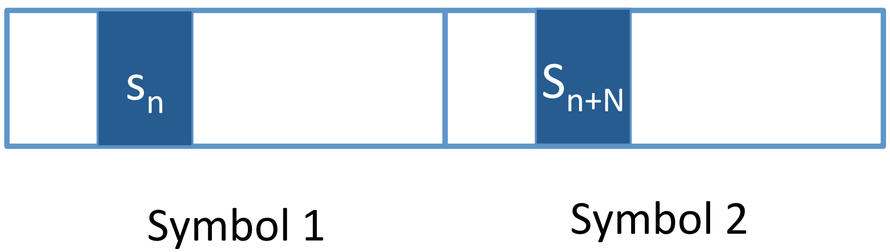
Carrier Frequency Offset (CFO)



- The oscillators of Tx and Rx are not typically tuned to identical frequencies
  - Up-convert baseband signal s_n to passband signal $y_n = s_n * e^{j2\pi f_{tx} n T_s}$
  - Down-convert passband signal y_n back to $r_n = s_n * e^{j2\pi f_{tx} n T_s} * e^{-j2\pi f_{rx} n T_s} = s_n * e^{j2\pi f_{\Delta} n T_s}$
  - Error accumulates

Correct CFO in Time Domain

$r_n = s_n * e^{j2\pi \mathbf{f_\Delta n} T_s}$ $\quad$ $r_{n+N} = s_{n+N} * e^{j2\pi \mathbf{f_\Delta (n+N)} T_s}$



$$z = \sum_{n=1}^{L} r_n r_{n+N}^*$$

$$= \sum_{n=1}^{L} e^{-j2\pi f_\Delta N T_s} s_n s_{n+N}^*$$

$$= e^{-j2\pi f_\Delta N T_s} \sum_{n=1}^{L} \left| s_n \right|^2$$

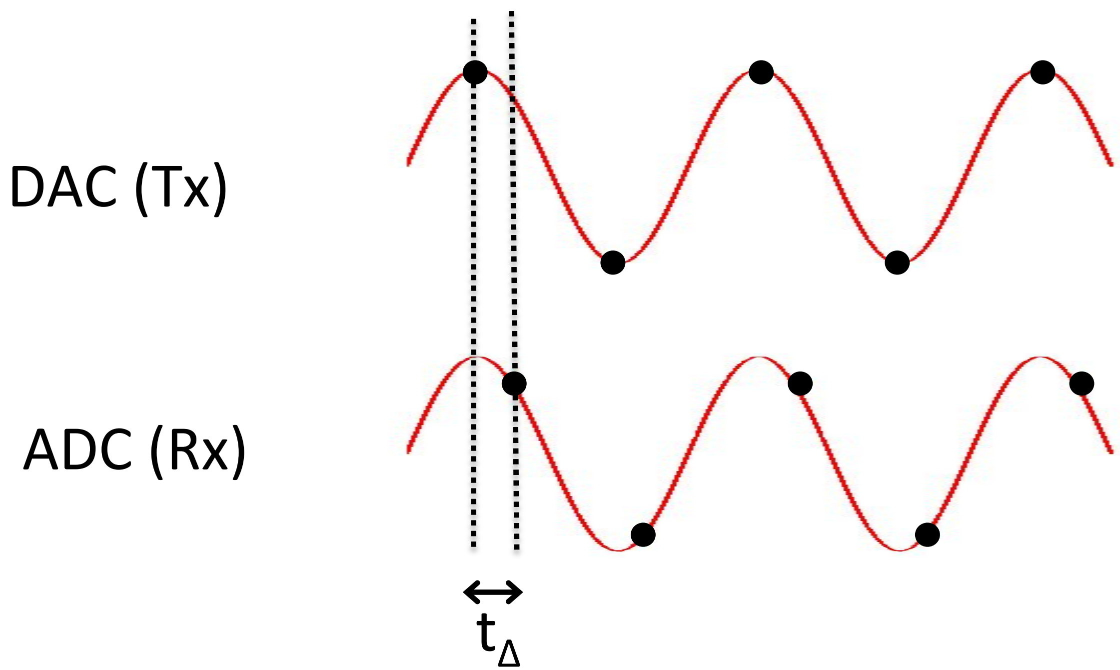
$$r_n r_{n+N}^* = s_n e^{j2\pi f_\Delta n T_s} s_{n+N}^* e^{-j2\pi f_\Delta (n+N) T_s}$$

$$= e^{-j2\pi f_\Delta N T_s} s_n s_{n+N}^*$$

$$= e^{-j2\pi f_\Delta N T_s} \left| s_n \right|^2$$

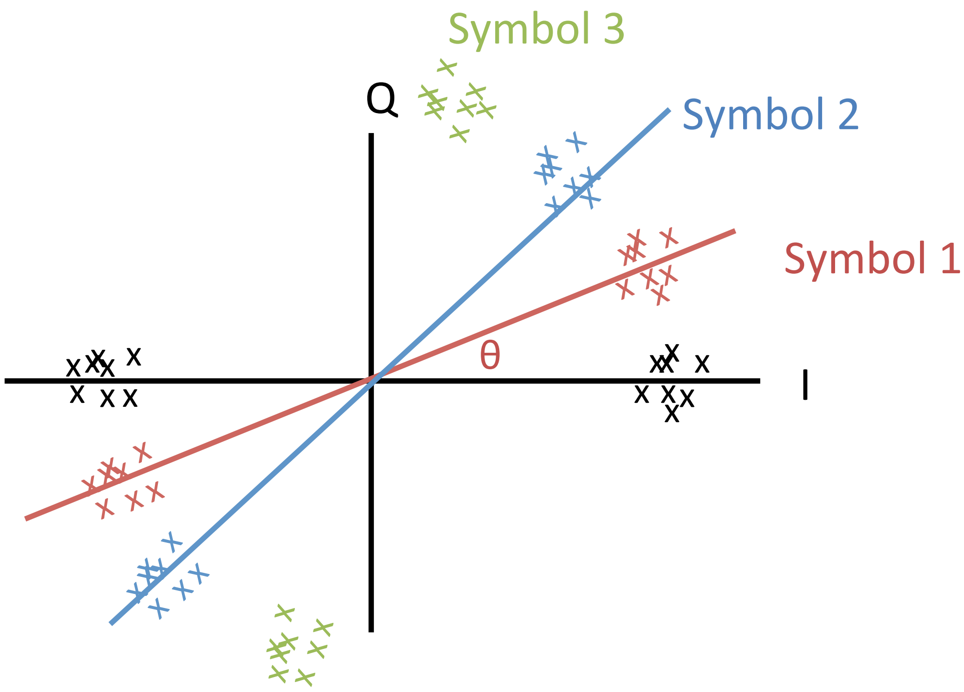
$$f_\Delta = \frac{1}{2\pi N T_s} \angle z$$

Sampling Frequency Offset (SFO)



- The transmitter and receiver may sample the signal at slightly different offset
  - Rotate the signal
- $Y_i=H_iX_i * e^{j2\pi t_\Delta i N_s/N_{fft}}$
- All subcarriers experience the same sampling delay, but have different frequencies

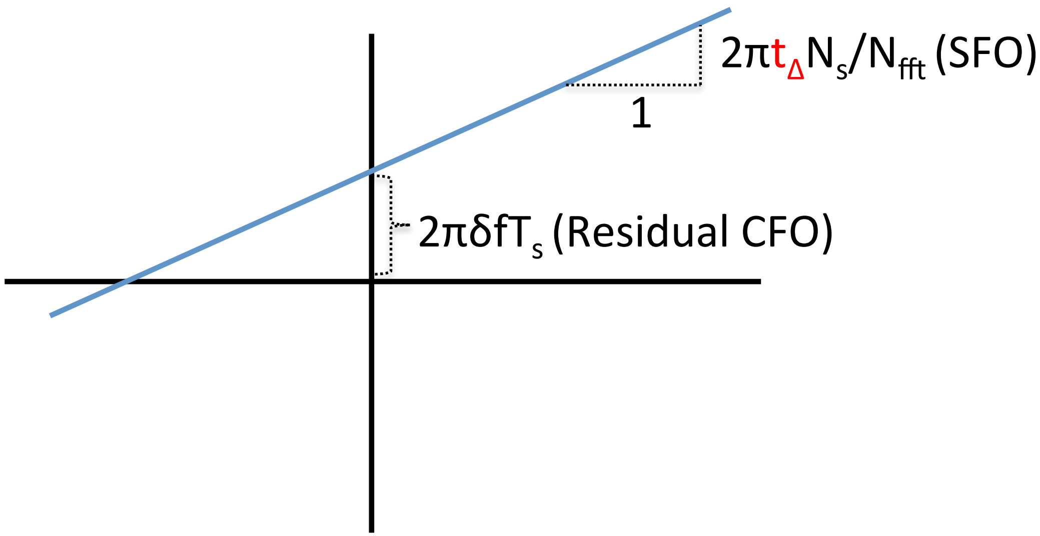
Sample Rotation due to SFO



Ideal BPSK signals (No rotation)

Signals keep rotating

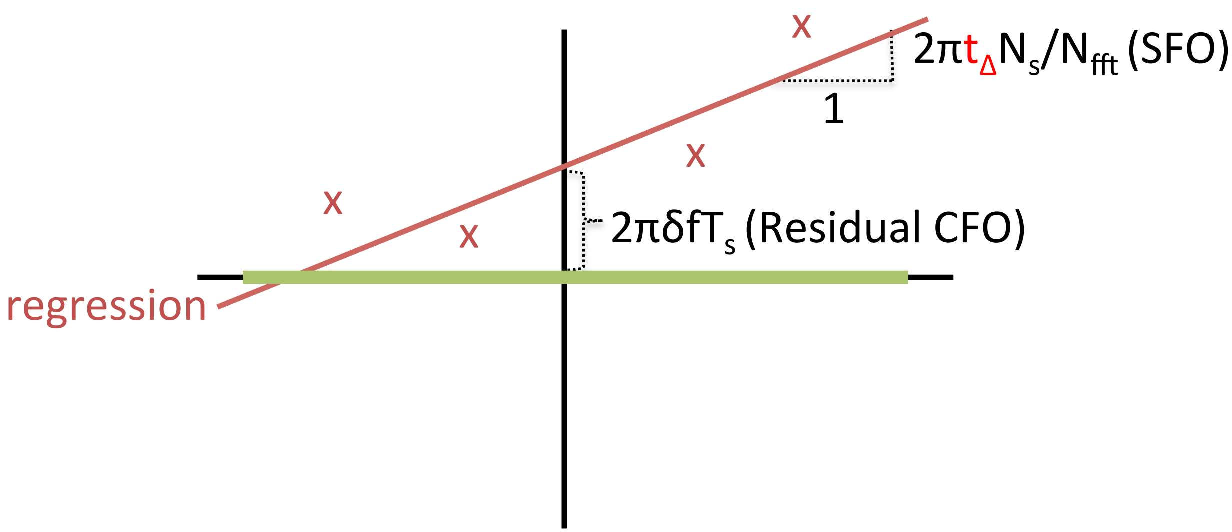
Correct SFO in Frequency Domain



Change in phase between Tx and Rx after CFO correction

- SFO: slop; residual CFO: intersection of y-axis

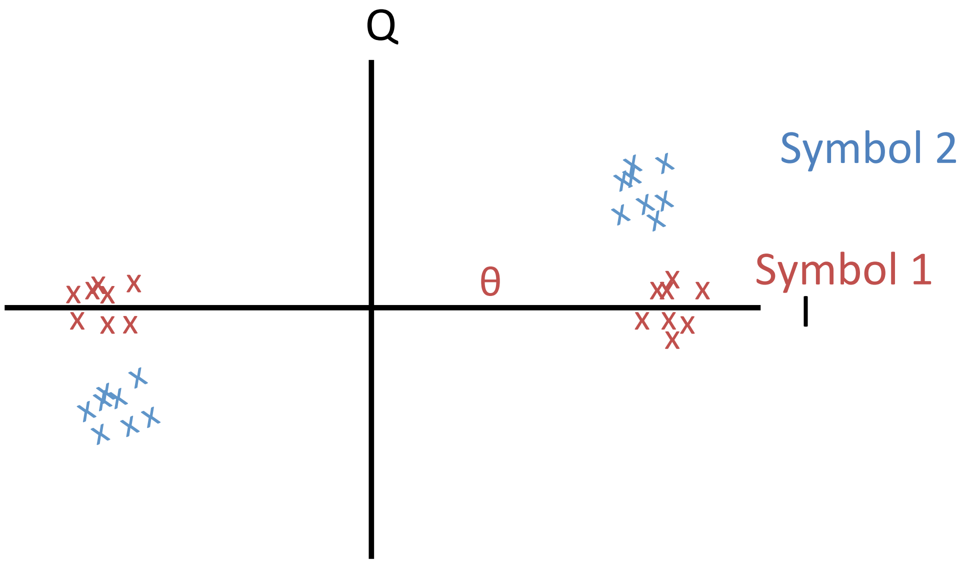
Data-aided Phase Tracking



Change in phase between Tx and Rx after CFO correction

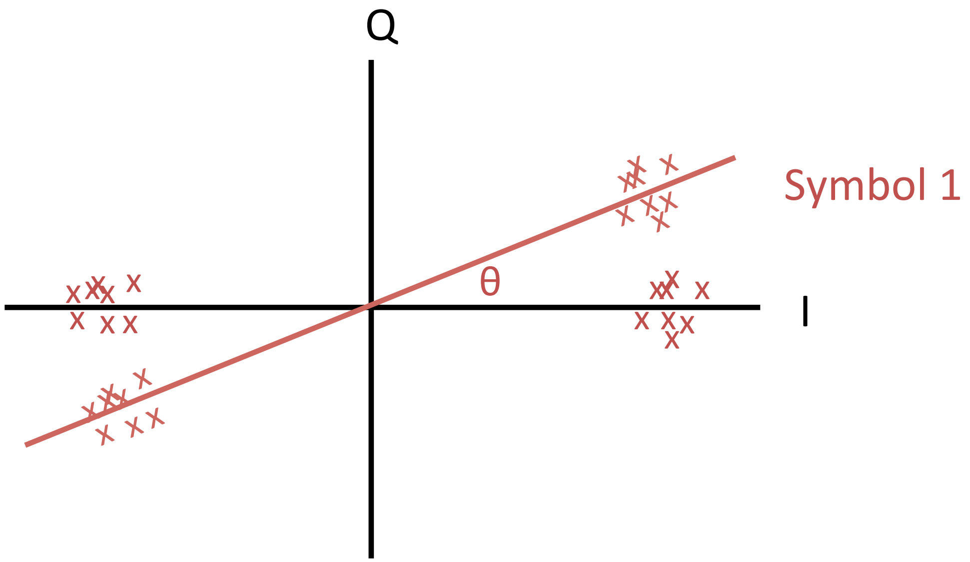
- Using pilot bits (known samples) to compute $H_i{}^*e^{j2\pi t\Delta i N_s/N_{fft}} = Y_{i}/X_i$
- Find the phase change experienced by the pilot bits using regression
- Update $H_l = H_i{}^*e^{j2\pi t\Delta i N_s/N_{fft}}$ for every symbol

After Phase Tracking



After correction

Nondata-aided Phase Tracking



OFDM Diagram